

**A METHOD OF DETERMINING PARAMETERS OF FORMATIONS  
THROUGH WHICH A BOREHOLE PASSES**

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5 The present invention relates to a method of determining parameters of formations through which a borehole passes, and more particularly to such a method of determining parameters on the basis of a resistivity log recorded in said borehole by means of a measuring and recording tool, said method comprising the steps consisting in:

- 10 • determining said formation parameters by a parameter inversion method so as to obtain a model of the formations;
- calculating the response of said tool to said model;
- using a comparison criterion for comparing said calculated response with said recorded log; and
- 15 • performing at least one new iteration if said comparison criterion is not satisfied.

Such methods are known. In general, they are implemented after a first stage of initializing parameters.

It is well known to make resistivity logs in boreholes by means of tools designed to measure the resistivity of the formations through which the borehole passes by establishing galvanic currents or eddy currents in the formations around the tool. Such tools give a set of resistivity values at each of the depths at which measurements are performed, the values applying to various distances from the axis of the borehole.

The relationship between the resistivity measurements performed in this way and the actual characteristics of the formations through which the borehole passes are typically affected by three types of effect:

- 25 a) the hole effect due to the presence of the borehole, which is generally filled with a drilling mud that is much more conductive than the formations;
- b) the shoulder bed effect due to the presence of generally heterogeneous formations above and below the zone being investigated, which formations can perfectly well have greater conductivity or greater resistivity than the formations level with the apparatus (it should be observed that the shoulder bed effect can be considerable even if the sonde is completely contained within a homogeneous bed surrounded by formations of different resistivities); and
- 30 c) the invasion effect due to the presence of drilling mud filtrate in the vicinity of the borehole, where the filtrate has replaced some of the fluids that were initially present in the formations.

To a first approximation, the invaded zone is represented by a region extending between the borehole and a cylinder of diameter  $d_i$  that is coaxial with the borehole and that has radially uniform resistivity  $R_{x0}$ , and beyond which a virgin zone of resistivity  $R_t$  is to be found. That model has three unknowns:  $R_t$ ,  $R_{x0}$ , and  $d_i$ , which is why at least three measurements having different radial investigation depths are recorded simultaneously so as to be able to determine the three unknowns.

The problem thus consists in determining at each depth a set of geometrical and electrical parameters, for example in this case the invasion diameter  $d_i$  and the resistivity  $R_{x0}$  of the invaded zone and the resistivity  $R_t$  of the virgin zones, on the basis of a set of resistivity measurements performed at depths that are separated by a given measurement pitch, the resistivities measured at each depth each being characteristic of the resistivity of the formation at a certain distance from the axis of the borehole.

Various methods are known that are capable of resolving that problem. Such methods are essentially of three types.

A first type of known method makes use of deconvolution filters. That approach assumes that the observed signal is the result of convolution between the real distribution of resistivities and a filter which represents the response of the tool to a resistivity distribution (or a conductivity distribution if the convolution is performed on the basis of a conductivity distribution). The method of interpretation then consists in deconvoluting the observed log by means of a known filter so as to discover the resistivity distribution. This step corresponds to correcting the initial data for shoulder bed effects.

This step is preceded or followed by a radial correction step seeking to correct for the results of the invasion effect.

The convolution filter can also be estimated by calculating the response of the tool to a formation possessing a resistivity distribution close to reality, which corresponds to local linearization of the filter.

Methods of that type are limited by the fact that the corrections performed therein for the shoulder bed effect and for the invasion effect are assumed to be independent whereas in reality they cannot, in general, be separated.

Another type of known method consists in partitioning the formation into cells.

Attempts are then made to obtain an image of the formation in terms of resistivity. The formation is partitioned into cells (usually in rectangle following the borehole axis and perpendicular to that axis), and a resistivity value is defined for each cell. A calculation algorithm is then used, e.g. using finite elements, finite differences, or a neural network, to calculate the response of the tool to the formations modelled in

this way, so as to determine whether the assumed resistivity distribution explains the apparent resistivity.

The horizontal boundaries of the cells can be determined by a segmentation algorithm and the vertical boundaries are often fixed by the user. In general, the use of  
 5 cells of large size leads to poor definition and thus to a poor approximation to reality, while the use of small cells can lead to instability which requires constraints to be fixed concerning the suddenness with which resistivity can vary, and such constraints deform the solution.

The unknowns of the inverse problem which are electrical unknowns only, are  
 10 in this case the resistivities of the various cells. These resistivities present a large number of degrees of freedom, that can lead to the above-mentioned instability. As mentioned above, one way of resolving such instability problems consists in imposing constraints, however such constraints suffer from the drawback of falsifying the solution.

15 The problem is easier to resolve when the number of resistivities describing a layer or bed is relatively small. However, under such circumstances, the model of the formation is less accurate. A bed split into a small number of radial zones does not accurately approximate the more realistic piston-profile model ( $R_{x0}$ ,  $d_t$ , and  $R_t$ ). This model insufficiency introduces a systematic bias into the inversion process.

20 A third type of known method consists in performing parametric local inversion. The general idea on which this approach is based is to use Newton's method to invert all of the unknown parameters describing the formation, such as the positions of the bed boundaries, and the values of  $d_i$ ,  $R_{x0}$ , and  $R_t$ , i.e. in this case the geometrical and resistivity parameters. In order to obtain a large enough number of  
 25 observable magnitudes, several readings of the tool near the area inverted can be used. A criterion – for instance a quadratic criterion – for evaluating error between observable magnitudes and reconstituted reading is minimized.

The advantage of that method is its flexibility. A wide variety of formation models can be used and there is a large amount of freedom in selecting input  
 30 measurements.

However, because of the generality of that approach, the inverse problem is often highly non-linear. Consequently, only local inversion can be performed and accurate gradients must be estimated, which is extremely expensive in terms of computation time.

35 Furthermore, when the method is used on real logging data, instabilities are observed which are the result of the local nature of the inversion.

The present invention seeks to mitigate those drawbacks.

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To this end, the invention provides a method of determining parameters of formations through which a borehole passes, on the basis of a resistivity log recorded in said borehole by means of a measuring and recording tool, the method comprising the steps consisting in determining said formation parameters by a parameter inversion  
5 method so as to obtain a model of the formations; calculating the response of said tool to said model; using a comparison criterion for comparing said calculated response with said recorded log; and performing at least one new iteration if said comparison criterion is not satisfied; said method being characterized by the fact that the step of determining said parameters from log data is performed by a quasi-Newton method;  
10 and said quasi-Newton method is implemented on pseudo-parameters that are homogeneous and that are determined from said formation parameters.

The method of the invention assumes that the boundaries between the layers (beds) are known.

In addition, any method known in the prior art is used for reconstituting the  
15 response of the tool to a given description of the formation (direct model).

For each bed, a bed model can be selected as being the most realistic. One particular bed model is a partition of the bed into various radial zones each of which possesses contact resistivity. In each bed, some of the parameters can be selected as being the unknowns of the inverse problem. Both resistivity and the positions of the  
20 radial interfaces constitute possible unknowns.

A certain number of "observables" are associated with each bed. An "observable" is a measurement made by the tool which is considered to be a function of the unknowns of the bed. For example, if the tool gives two measurements at each depth, the two measurements obtained close to the middle of the bed can be selected  
25 as being a function of two parameters of the bed model.

The observables can be the result of the same kind of measurement, but taken at different depths or with the tool in different positions. The rule for selecting observables depends on the tool and on the model that is to be inverted. The generality of the method of the invention makes inversion possible with a wide  
30 selection of observables.

Consequently, the invention is based on local methods of parametric inversion. Nevertheless, optimization is now performed that is not local, i.e. that takes place simultaneously over all of the layers.

The hole effect is obviated by assuming that the resistivity of the mud is  
35 known and also that the diameter of the borehole is known. In addition, the positions of the horizontal boundaries between the beds are known, e.g. from zero crossings of the second derivative of conductivity.

It is also assumed that the resistivity values at certain radial depths are known, e.g. the resistivities  $R_{LLs}$  and  $R_{LLd}$ , or other radial distributions of resistivity.

The unknowns at each depth are  $R_{x0}$ ,  $R_t$ , and  $d_i$ .

If it is desired to determine specifically  $R_{x0}$ , then it is necessary to know the microresistivity, assuming that the measured values are  $R_{LLs}$  and  $R_{LLd}$ .

In any event, it is necessary in theory for the number of unknowns to be smaller than the number of observables. In practice, the method of the invention requires that the number of unknowns be equal to the number of observables. When the number of observables is greater than the number of unknowns, either no account is taken of some of the observables, or else combinations of observables are taken as unknowns.

The invention thus consists essentially in:

- firstly applying a quasi-Newton method to the problem of parametric inversion; and
- secondly improving convergence in a quasi-Newton method which is generally affected by combining geometrical parameters such as  $d_i$  and electrical parameters such as  $R_{x0}$  and  $R_t$ , by using combinations of such geometrical and electrical parameters as the unknowns, e.g. combinations that all consist in pseudo-electrical magnitudes, and in particular pseudo-resistivities.

Consequently, two transformations are generally performed when implementing the invention.

A first transformation is generally performed in the observables space so as to reduce the number of observables to the number of unknowns. This is necessary in order to be able to apply the inversion algorithm which assumes that the number of observables is equal to the number of unknowns.

The second transformation is performed in the unknowns space to obtain a set of unknowns that are homogeneous, in particular that are electrical, on the basis of unknowns that are not homogeneous, e.g. unknowns that are geometrical and unknowns that are electrical.

A particular implementation of the invention is described below by way of non-limiting example and with reference to the accompanying diagrammatic drawings, in which:

- Figure 1 is an overall flow chart of a method of the invention; and
- Figure 2 is a flow chart of an iteration of the calculation algorithm.

As shown in Figure 1, implementation of the method of the invention begins by a step 1 of measuring various physical magnitudes of the formation from inside a borehole. At each measurement level the number of magnitudes measured in this way

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must be not less than the number of parameters for which it is desired to obtain a value at each level.

By way of example, in the common case where the parameters that it is desired to determine are the resistivity values  $R_{x0}$  and  $R_t$ , and also the invasion distance  $d_i$ , making a total of three parameters, at least three magnitudes will be measured at each step.

By way of example, it is possible to measure the magnitudes  $R_{LLs}$  and  $R_{LLd}$  by means of a tool known as a Dual Laterolog (Schlumberger registered trademark) and to measure  $R_{x0}$  directly by means of a device known as an MSFL (Schlumberger trademark for "micro spherically focused log"). In another implementation of this step of the method, it is possible to measure the resistivities  $R_{LA1}$ , ...,  $R_{LA5}$  obtained by means of a device known as an HRLA (Schlumberger trademark for: "high resolution laterolog array"), which correspond to formation resistivities as measured at increasing distances from the axis of the borehole.

Following step 2 consists in determining the locations of the beds or layers whose parameters are to be determined. The locations of the beds can be determined automatically by means of a segmentation algorithm of known type, e.g. based on looking for points of inflection in the apparent conductivity values.

During this step, a first initialization value is given to each parameter.

Following step 3 consists in selecting the appropriate number of "observable" magnitudes. Remaining in the context where it is desired that  $R_{x0}$ ,  $R_t$ , and  $d_i$  should be determined, the observables used will be the three measurements obtained by means of the dual laterolog and the MSFL, in the first case.

In the second case where measurements are made using the HRLA, one or two of the values  $R_{LA1}$ , ...,  $R_{LA5}$  can be eliminated and/or three observables can be obtained by combining these values, in particular by combining them in linear manner.

During following step 4, a value is given to each observable for each determined layer, as mentioned above.

It will be observed that more measurement steps are available than there are layers since it is necessary in particular to have at least three measurement steps in order to be able to determine a point of inflection.

For this purpose, it is possible to interpolate the measurements performed within a layer so as to obtain the value of each of the observables in the middle of the layer.

It is also possible to give each layer the values of the observables as measured at the measurement point that is closest to the middle of the layer.

The following step 5 consists in determining the looked-for parameters.

Mathematically, the problem is thus posed as follows.

This problem is the inverse of the direct problem in which it is desired to determine the response of the measurement tools, the observables of the problem of the invention, from the actual parameters of the formation, i.e. the unknowns in the problem of the invention.

We use the notation:

$f(U)=O$  for the direct problem; and

$U=f^{-1}(O)$  for the problem of the invention, i.e. the inverse of the direct problem.

The direct problem can be resolved, i.e. the system of non-linear equations  $f_i$  can be evaluated as a function of the layer model and of previously selected observables. By way of example, when the physical characteristics under consideration are resistivities, the layer model consists in defining how the resistivity of the terrain varies as a function of distance from the axis of the borehole, or indeed the  $R_{x0}$ ,  $R_t$ , and  $d_i$  model.

To solve the inverse problem, it is necessary to use an iterative method, i.e. to determine a stream of values  $U_1, U_2, \dots, U_\infty$  tending towards the solution  $U_{\text{solution}}$ , such that:

$$f(U_{\text{solution}}) = O_{\text{observed}}$$

In conventional methods, a term  $U_+$  of the above stream is obtained from the current term  $U_c$  using the following equation:

$$(1) \quad U_+ = U_c - (\nabla f)^{-1} * [f(U_c) - O_{\text{observed}}]$$

where  $\nabla f$  is the Jacobian of the function  $f(U)$ .

It is recalled that the Jacobian of the function  $f$  is defined as follows:

$$\begin{aligned} \text{If } f: \mathcal{R}^N &\rightarrow \mathcal{R}^N \\ \text{then } \nabla f: \mathcal{R}^{N \times N} &\rightarrow \mathcal{R}^{N \times N} \\ \text{with } (\nabla f)_{i,j} &= \frac{\partial f_i}{\partial u_j} \end{aligned}$$

It can thus be seen that a large number of evaluations of the function  $f$  is required to estimate the  $N \times N$  matrix, given that the system of equations can typically have 1000 observables and 1000 unknowns.

According to the invention, instead of using Newton's method in which, on each iteration, a new vector  $U$  is estimated and a new Jacobian is calculated, a quasi-Newton method is used in which both the vector  $U$  and the Jacobian are estimated at each step. If the current estimate of the Jacobian is  $B_c$ , then equation (1) is replaced by:

$$(2) \quad U_+ = U_c - B_c^{-1} * [f(U_c) - O_{\text{observed}}]$$

A new estimate of the Jacobian can be obtained on each iteration, for example, using the Broyden method as described at pages 113 to 131 of "Iterative method for linear and non-linear equations" by C.T. Kelley published by Society for Industrial and Applied Mathematics.

More precisely, the matrix  $B_+^{-1}$  at the following step is obtained from the current matrix  $B_c^{-1}$  using the following equation:

$$(3) \quad B_+^{-1} = B_c^{-1} * \left( I + \frac{[f(U_c) - O_{\text{observed}}] * s^T}{s^T * s} \right)$$

where  $I$  is the identity matrix and where  $s = U_+ - U_c$ .

With this method, it is therefore no longer necessary to know the Jacobian.

Nevertheless, to ensure that the stream of  $B_n^{-1}$  converges quickly, the initial data is preconditioned so as to be homogeneous. In the present case, this is achieved not by using the variables  $R_{x0}$ ,  $R_t$ , and  $d_i$ , which are not homogeneous, given the presence of  $d_i$ , but by using the resistivities  $R_{x0}$  and  $R_t$ , together with the pseudo-resistivity  $R_a$  as defined by:

$$(4) \quad \begin{cases} R_a = \alpha * R_{x0} + (1 - \alpha) * R_t \\ \alpha = \frac{d_{imax} - d_i}{d_{imax} - d_{imin}} \end{cases}$$

where  $d_{imax}$  and  $d_{imin}$  are parameters fixed so as to correspond to maximum and minimum acceptable values for  $d_i$ .

Figure 2 is a complete flow chart for one interaction.

For example, the starting point is an identity matrix, and at 11, an iteration of the above-described quasi-Newton method is applied to obtain pseudo-resistivities  $R_1$ ,  $R_2$ , and  $R_3$  for each layer, as shown as 12.

The pseudo-resistivities  $R_1$ ,  $R_2$  and  $R_3$  are then transformed at 13 into physical magnitudes 14:  $R_{x0}$ ,  $R_t$ , and  $d_i$ .

It is then possible at 15 to update each layer of the model of the formation so as to obtain a formation model 16 which is updated in terms of  $R_{x0}$ ,  $R_t$  and  $d_i$ .



The response of the tool to the formation 16 is then calculated at 17 in conventional manner, e.g. by a finite element method, so as to obtain a simulated log 18.

In step 19, the simulated log is compared with a real log 20. If the difference  
5 between the simulated log and the real log satisfies a predetermined criterion, the iteration terminates at 21. Otherwise, a new iteration is performed.